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A Mixed Problem for a Wave Equation with a Nonzero Initial Velocity

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We study a mixed problem for the wave equation with a continuous complex potential in the case of a nonzero initial velocity $u_t(x,0) = \psi(x)$ and two types of two-point boundary conditions: the ends are fixed and when each of the boundary boundary conditions contains a derivative with respect to x. A classical solution in the case $\psi(x) \in W_2^1[0,1]$ is obtained by the Fourier method with respect to the acceleration of the convergence of Fourier series by the resolvent approach with the help of A. N. Krylov's recommendations (the equation is satisfied almost everywhere). It is also shown that in the case when $\psi(x) \in L[0,1]$ the series of a formal solution for a problem with fixed ends converges uniformly in any bounded domain, and for the second problem it converges only everywhere and for both problems is a generalized solution in the uniform metric.

Key words: wave equation, formal solution, spectral problem, resolvent.

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