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Almost Periodic at Infinity Functions Relative to the Subspace of Functions Integrally Decrease at Infinity

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In the paper we introduce and study a new class of almost periodic at infinity functions, which is defined by means of a subspace of integrally decreasing at infinity functions. It is wider than the class of almost periodic at infinity functions introduced in the papers of A. G. Baskakov (with respect to the subspace of functions vanishing at infinity). It suffices to turn to the approximation theory for a new class of functions, where the Fourier coefficients are slowly varying at infinity functions with respect to the subspace of functions that decrease integrally at infinity. Three equivalent definitions of functions almost periodic at infinity with respect to integrally decreasing functions at infinity are formulated. For their investigation, the theory of Banach modules over the algebra $L^1(\mathbb{R})$ of summable functions is applied. Almost periodic functions at infinity appear naturally as a solution of differential equations. Criteria for the almost periodicity at infinity of bounded solutions of ordinary differential equations of the form $\dot{x}(t) = Ax(t) + z(t), t \in \mathbb{J}$ are formulated, where A is a linear operator and z is an integrally decreasing function at infinity, defined on infinite interval $\mathbb{J}$ that coincides with one of the sets $\mathbb{R}$ or $\mathbb{R}_+$.

Key words: almost periodic at infinity functions, slowly varying at infinity functions, integral decreasing at infinity functions.

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