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On an Inner Estimate of a Convex Body by the Lebesgue Set of Convex Differentiable Function

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A finite-dimentional problem of embedding the largest by the inclusion of lower Lebesgue set of given convex function $f(x)$ in a given convex body $D \subset \mathbb{R}^p$ is considered. This problem is the generalization of the problem of inscribed ball (function $f(x)$ is some norm, and the Lebesgue sets are the corresponding balls). The function $f(x)$ must be differentiable on $\mathbb{R}^p$ possibly expending the point 0_p and 0_p is the uniqueness point of minimum. Mathematical formalization of this problem is proposed in the form of finding maximin of a function of the difference of arguments. It is proved that the objective function of this maximin problem is Lipschitzian on all space $\mathbb{R}^p$ and quasiconcave on the set D. Also, superdifferentiability (in the sense of V. F. Demyanov – A. M. Rubinov) of objective function on the interior of D is established and the corresponding formula of superdifferential is derived. The necessary and sufficient solution conditions and the condition for uniqueness of solution are obtained on the basis of this formula of superdifferential.

Key words: convex body, inner estimate, minimax, supperdifferential, quasiconcave function.

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