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Well-posedness of the Dirichlet Problem for One Class of Degenerate Multi-dimensional Hyperbolic-parabolic Equations

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It has been shown by Hadamard that one of the fundamental problems of mathematical physics, the analysis of the behavior of oscillating string is an ill-posed problem when the boundary-value conditions are imposed on the entire boudary of the domain. As noted by A. V. Bitsadze and A. M. Nakhushev, the Dirichlet problem is ill-posed not only for the wave equation but for hyperbolic PDEs in general. This author has earlier studied the Dirichlet problem for multi-dimensional hyperbolic PDEs, where he has shown that the well-posedness of this problem crucially depends on the height of the analyzed cylindric domain. This paper, using the method developed in the authors previous papers, shows the unique solvability (and obtains an explicit form of the classical solution) of the Dirichlet problem in the cylindric domain for one class of degenerate multi-dimensional hyperbolic-parabolic equations. We also obtain a criterion for the uniqueness of the solution.

Key words: well-posedness, Dirichlet problems, degenerate equations, criterion, Bessel function.

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References

1. Nakhushev A. M. *Zadachi so smeshcheniem dlia uravneniia v chastnykh proizvodnykh* [Problems with Shift for Partial Differential Equations]. Moscow, Nauka, 2006. 287 p. (in Russian).
2. Vragov V. N. *Kraevye zadachi dlia neklassicheskikh uravnenii matematicheskoi fiziki* [Boundary-Value Problems for Nonclassical Equations of Mathematical Physics]. Novosibirsk, Novosibirsk State Univ. Press, 1983. 84 p. (in Russian).
3. Karatoprakliev G. D. Boundary Value Problems for Mixed Type Equations in Multi-dimensional Domains. *Partial Differential Equations Banach Center Publications*, 1983, vol. 10, pp. 261–269 (in Russian).
4. Aldashev S. A. Well-posedness of Dirichlet problem for one class of multi-dimensional hyperbolic-parabolic equations. *Ukrainskii Matematicheskii Vestnik*, 2013, vol. 10, no. 2, pp. 147–157 (in Russian).
5. Aldashev S. A. Correctness of the Dirichlet Problem for a Class of Multidimensional Hyperbolic-Parabolic Equations. *J. Math. Sci.*, 2013, vol. 194, iss. 5, pp. 491–498. DOI: 10.1007/s10958-013-1542-z.
6. Aldashev S. A. Well-posedness of the Dirichlet and Poincare problems for a multidimensional Gellerstedt equation in a cylindrical domain. *Ukr. Math. J.*, 2012, vol. 64, iss. 3, pp. 484–490. DOI: 10.1007/s11253-012-0660-y.
7. Aldashev S. A. Well-posedness of Dirichlet and Poincare problems in a cylindrical domain for degenerate many dimensional hyperbolic equations with Gellerstedt operator. *Nonlinear Oscilations*, 2015, vol. 18, no. 1, pp. 10–19 (in Russian).
8. Mikhlin S. G. *Multidimensional singular integrals and integral equations*. Oxford, New York, Paris, Pergamon Press, 1965. 255 p. (Russ. ed. : Moscow, Fizmatgiz, 1962. 254 p.)

9. Kamke E. *Spravochnik po obyknovennym differentsial'nym uravneniiam* [Manual of ordinary differential equations]. Moscow, Nauka, 1965. 703 p. (in Russian).
10. Beitmen G., Erdeii A. *Vysshie transtsendentnye funktsii* [Higher Transcendental Functions]. Moscow, Nauka, 1974, vol. 2. 297 p. (in Russian).
11. Kolmogorov A., Fomin S. *Elements of the Theory of Functions and Functional Analysis*. Mineola, New York, USA, Dover Publ., 1999. 288 p. (Russ. ed. : Moscow, Nauka, 1976. 543 p.)
12. Tikhonov A. N., Samarskii A. A. Equations of mathematical physics. New York, Dover, 1990. 800 p. (Russ. ed. : Moscow, Nauka, 1966. 724 p.)
13. Smirnov V. I. *Kurs vysshei matematiki* [Higher Mathematics Course]. Moscow, Nauka, 1981, vol. 4, pt. 2. 550 p. (in Russian)
14. Fridman A. *Uravneniia s chastnymi proizvodnymi parabolicheskogo tipa* [Partial differential Equations of parabolic type]. Moscow, Mir, 1968. 527 p. (in Russian).
15. Aldashev S. A. The Well-Posedness of the Dirichlet Problem for Degenerate Multi-Dimensional Hyperbolic-Parabolic Eguation. *News of the National Academy of Sciences of the Republic of Kazakhstan. Physico-Mathematical Ser.*, 2014, vol. 5, no. 297, pp. 7–11 (in Russian).

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